INSA 3BIM - ODE and Modelling
Friday 13 Decembre 2019

Instructions

This form will be analyzed by optical reading, so please strictly respect the following rules:

- To check an answer, fill in the ☐ in black (■) by using a black pen;
- To correct an answer, erase the ■ with white correcting fluid; there is no need to redo the ☐;
- Do not write anything either in the header nor in margins;
- Symbol ♣ stands for multiple answers; note that the number of correct answers is indeterminate (0, 1, 2,. . .). Absence of this sympbol means that the question has a unique correct answer.

Multiple choice questions have a null mean: correct answer = 1 point ; no answer = 0 point ; wrong answer to a question with n proposals = $-\frac{1}{n-1}$ points.

A private mémorandum hand-written on both sides of a page, A4 format, is allowed, as well as any type of calculator **not** connected to the Internet. The use of the smartphone is stricly forbidden.

Identity

Fill in empty fields below and encode your student number beside.

Last and first name:

. . . .

Student number:

. . . .

☐0	☐0	☐0	☐0	☐0	☐0	☐0
☐1	☐1	☐1	☐1	☐1	☐1	☐1
☐2	☐2	☐2	☐2	☐2	☐2	☐2
☐3	☐3	☐3	☐3	☐3	☐3	☐3
☐4	☐4	☐4	☐4	☐4	☐4	☐4
☐5	☐5	☐5	☐5	☐5	☐5	☐5
☐6	☐6	☐6	☐6	☐6	☐6	☐6
☐7	☐7	☐7	☐7	☐7	☐7	☐7
☐8	☐8	☐8	☐8	☐8	☐8	☐8
☐9	☐9	☐9	☐9	☐9	☐9	☐9

Question 1 ♣ Among the following systems, which ones correspond to linear systems?

☐ $\begin{cases} \dot{x} = x(1+2y) \\ \dot{y} = y(1+3x) \end{cases}$

☐ $\begin{cases} \dot{x} = x - x^2 - xy \\ \dot{y} = -y + xy \end{cases}$

■ $\begin{cases} \dot{x} = 4x - 8y \\ \dot{y} = 3x - 4y \end{cases}$

■ $\begin{cases} \dot{x} = x - y \\ \dot{y} = y - x \end{cases}$

Let consider dynamical system $(S_1)\,\dot{\mathbf{X}} = \mathbf{A}\mathbf{X}$ with $\mathbf{X}=\ (x(t), y(t))$ and $\mathbf{A}$ a sqaure matrix of dimension 2.

Question 2 ♣ Which condition(s) is(are) required to make system (S_1) having a unique equilibrium point?

■ $\mathbf{A}$ can be rendered diagonal

■ $\mathbf{A}$ is a transfer matrix

☐ $\mathbf{A}$ has a null eigenvalue

☐ $\det \mathbf{A} = \mathbf{0}$

■ $\det \mathbf{A} \neq \mathbf{0}$

■ $\mathbf{A}$ is regular

Question 3 How writes the characteristic polynomial of matrix $\mathbf{A}$ in (S_1)?

- ☐ $\lambda^2 + \text{tr}(\mathbf{A})\lambda + \det(\mathbf{A}) = \mathbf{0}$
- ☐ $\lambda^2 - \text{tr}(\mathbf{A})\lambda - \det(\mathbf{A}) = \mathbf{0}$
- ☐ $\lambda^2 + \text{tr}(\mathbf{A})\lambda - \det(\mathbf{A}) = \mathbf{0}$
- ■ $\lambda^2 - \text{tr}(\mathbf{A})\lambda + \det(\mathbf{A}) = \mathbf{0}$

Question 4 ♣ If system (S_1) has a unique equilibrium point, when is it unstable?

- ☐ $\det(\mathbf{A}) < \mathbf{0}$
- ☐ $\text{tr}(\mathbf{A}) < \mathbf{0}$
- ■ $\text{tr}(\mathbf{A}) > \mathbf{0}$
- ■ $\det(\mathbf{A}) > \mathbf{0}$
- ☐ $\Delta < 0$
- ☐ $\Delta > 0$

Question 5 If $\det \mathbf{A} \neq \mathbf{0}$ in system (S_1), how writes the general solution for the initial condition X_0?

- ☐ $X(t) = X_0 e^{t\mathbf{A}}$
- ■ $X(t) = e^{t\mathbf{A}} X_0$
- ☐ $X(t) = X_0 e^{-t\mathbf{A}}$
- ☐ $X(t) = e^{-t\mathbf{A}} X_0$

Question 6 Let $\mathbf{A} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$. How writes $e^{t\mathbf{A}}$?

- ☐ $\begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix}$
- ■ $\begin{pmatrix} e^{-t} & 0 \\ 0 & e^t \end{pmatrix}$
- ☐ $\begin{pmatrix} e^t & te^t \\ 0 & e^t \end{pmatrix}$
- ☐ $e^t \begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix}$

Question 7 Let $(S_2) : \begin{cases} \dot{x} = 4x - 8y \\ \dot{y} = 3x - 4y \end{cases}$. What is the nature of the equilibrium point?

- ☐ Asymptotically stable star
- ☐ Saddle node
- ☐ Unstable spiral
- ■ Center
- ☐ Asymptotically stable node
- ☐ Unstable degenerated node

Question 8 ♣ Among the following matrices, which ones are in Jordan's format?

- ☐ $\begin{pmatrix} 4 & -8 \\ 8 & -4 \end{pmatrix}$
- ☐ $\begin{pmatrix} 4 & 0 \\ 1 & 4 \end{pmatrix}$
- ■ $\begin{pmatrix} 4 & -8 \\ 8 & 4 \end{pmatrix}$
- ■ $\begin{pmatrix} -4 & 1 \\ 0 & -4 \end{pmatrix}$
- ☐ $\begin{pmatrix} 4 & -8 \\ -8 & 4 \end{pmatrix}$
- ☐ $\begin{pmatrix} 4 & -8 \\ 3 & -4 \end{pmatrix}$

Question 9 Let $\mathbf{A} = \begin{pmatrix} -6 & 0 \\ 0 & 0 \end{pmatrix}$. What is the topological equivalence class of $\dot{\mathbf{X}} = \mathbf{AX}$?

- ☐ A flux
- ■ A valley
- ☐ A crest
- ☐ A center

Let $(S_3) : \begin{cases} \dot{x} = x - x^2 - xy \\ \dot{y} = -y + xy \end{cases}$ describing the dynamics of two interacting species.

Question 10 What is the type of ecological interaction that system (S_3) is modelling?

- ■ prey-predator
- ☐ symbiosis
- ☐ competition
- ☐ commensalism

Question 11 Give the equilibrium points of (S_3).

- ☐ $(x_1^*, y_1^*) = (0,0)$ et $(x_2^*, y_2^*) = (0,1)$
- ☐ $(x_1^*, y_1^*) = (1,0)$ et $(x_2^*, y_2^*) = (1,1)$
- ☐ $(x_1^*, y_1^*) = (0,0)$ et $(x_2^*, y_2^*) = (1,-1)$
- ■ $(x_1^*, y_1^*) = (0,0)$, $(x_2^*, y_2^*) = (1,0)$

Question 12 How writes the jacobian matrix associated to (S_3)?

■ $\mathbf{A} = \begin{pmatrix} 1-2x-y & -x \\ y & -1+x \end{pmatrix}$

□ $\mathbf{A} = \begin{pmatrix} y & -1+x \\ 1-2x-y & -x \end{pmatrix}$

□ $\mathbf{A} = \begin{pmatrix} y & 1-2x-y \\ -1+x & -x \end{pmatrix}$

□ $\mathbf{A} = \begin{pmatrix} 1-2x-y & y \\ -x & -1+x \end{pmatrix}$

Question 13 How writes the jacobian matrix of (S_3) at the equilibrium point (x_1^*, y_1^*)?

■ $\mathbf{A}_1^* = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

□ $\mathbf{A}_1^* = \begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix}$

□ $\mathbf{A}_1^* = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

□ $\mathbf{A}_1^* = \begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$

Question 14 What is the nature of the equilibrium point (x_1^*, y_1^*) de (S_3)?

□ It is asymptotically stable

□ It is unstable

■ This is a saddle node

Question 15 Equilibrium point (x_2^*, y_2^*) de (S_3) is non hyperbolic.

■ True

□ False

Question 16 ♣ Among the following relationships, which ones are true?

■ $x = r\cos\theta$ et $y = r\sin\theta$

□ $r^2 = x^2 - y^2$

■ $\tan\theta = \frac{y}{x}$

□ $x = r\sin\theta$ et $y = r\cos\theta$

□ $\tan\theta = \frac{x}{y}$

■ $r^2 = x^2 + y^2$

Let $(S_4) : \begin{cases} \dot{x} = x - xy \\ \dot{y} = -y + xy \end{cases}$.

Question 17 ♣ The jacobian matrix at the equlibrium point origin in system(S_4):

■ always corresponds to a saddle node

□ corresponds to centers

■ is the matrix of the linear part of (S_4)

Question 18 ♣ Which ones of the following functions is a first integral for (S_4)?

□ $H(x,y) = \frac{x^2}{y} - \ln x - \ln y$

■ $H(x,y) = -\ln x - \ln y + x + y$

□ $H(x,y) = \ln(xy) - xy$

■ $H(x,y) = \ln x + \ln y - x - y$

Question 19 ♣ Which condition(s) on $H(x,y)$ is(are) required to demonstrate the existence of centers in (S_4)?

□ $H(x,y)$ is a continuous function

□ $H(x,y)$ is an homeomorphism

■ $H(x,y)$ has closed level curves

■ $H(x,y)$ is constant along the trajectories

Question 20 The positive quadrant is positively invariant for (S_4)?

☐ No

■ Yes