

Instructions

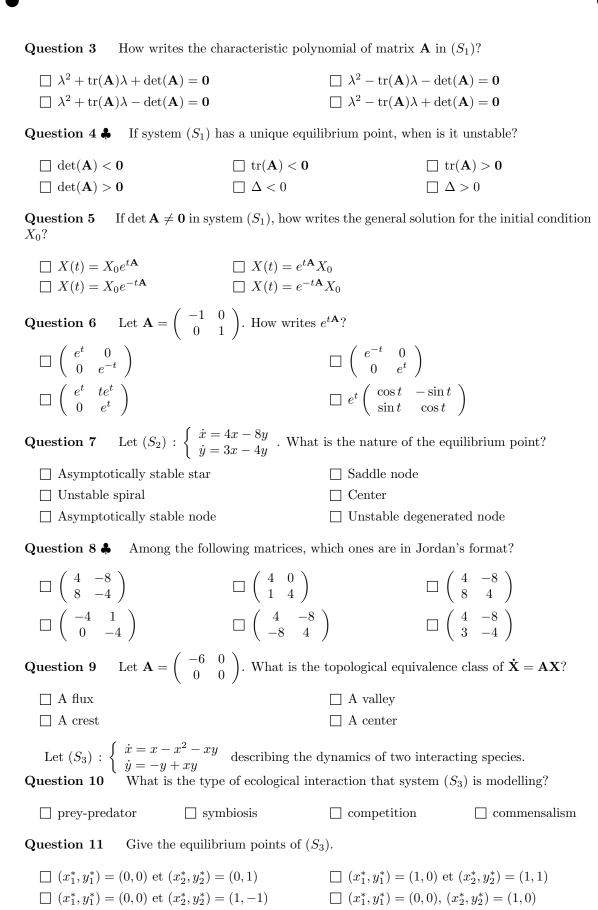
This form will be analyzed by optical reading, so please strictly respect the following rules:

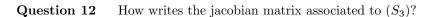
- To check an answer, fill in the □ in black (■) by using a black pen;
- To correct an answer, erase the \blacksquare with white correcting fluid; there is no need to redo the \square ;
- Do not write anything either in the header nor in margins;
- Symbol & stands for multiple answers; note that the number of correct answers is indeterminate (0, 1, 2,...). Absence of this sympbol means that the question has a unique correct answer.

Multiple choice questions have a null mean: correct answer = 1 point; no answer = 0 point; wrong answer to a question with n proposals = $-\frac{1}{n-1}$ points.

A private mémorandum hand-written on both sides of a page, A4 format, is allowed, as well as any type of calculator **not** connected to the Internet. The use of the smartphone is stricly forbidden.

| Identity | |
|---|---|
| Fill in empty fields below and encode your student number beside. | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ |
| Last and first name: Student number: | 3 3 3 3 3 3 4 4 4 4 4 4 4 4 5 5 5 5 5 5 5 5 6 6 6 6 6 6 6 6 6 |
| | \[\begin{array}{c c c c c c c c c c c c c c c c c c c |
| Question 1 Among the following systems, where the following systems is a system of the following systems. | hich ones correspond to linear systems? |
| $\square \left\{ \begin{array}{l} \dot{x} = x(1+2y) \\ \dot{y} = y(1+3x) \end{array} \right. \square \left\{ \begin{array}{l} \dot{x} = x-x \\ \dot{y} = -y + 1 \end{array} \right.$ | $x^2 - xy + xy$ |
| $\square \left\{ \begin{array}{l} \dot{x} = 4x - 8y \\ \dot{y} = 3x - 4y \end{array} \right. \square \left\{ \begin{array}{l} \dot{x} = x - 3y \\ \dot{y} = y - 3y \end{array} \right.$ | $y \\ v$ |
| Let consider dynamical system $(S_1)\dot{\mathbf{X}} = \mathbf{A}\mathbf{X}$ widimension 2. | th $\mathbf{X}=(x(t),y(t))$ and \mathbf{A} a square matrix of |
| Question 2 ♣ Which condition(s) is(are) require librium point? | red to make system (S_1) having a unique equi- |
| \square A can be rendered diagonal | ☐ A is a transfer matrix |
| ☐ A has a null eigenvalue | $\square \det \mathbf{A} = 0$ |
| $\square \det \mathbf{A} \neq 0$ | \square A is regular |





$$\square \mathbf{A} = \begin{pmatrix} 1 - 2x - y & -x \\ y & -1 + x \end{pmatrix}$$

$$\square \mathbf{A} = \begin{pmatrix} y & -1+x \\ 1-2x-y & -x \end{pmatrix}$$

$$\square \mathbf{A} = \begin{pmatrix} y & 1 - 2x - y \\ -1 + x & -x \end{pmatrix}$$

$$\square \mathbf{A} = \begin{pmatrix} 1 - 2x - y & y \\ -x & -1 + x \end{pmatrix}$$

Question 13 How writes the jacobian matrix of (S_3) at the equilibrium point (x_1^*, y_1^*) ?

$$\square \mathbf{A}_1^* = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\square \mathbf{A}_1^* = \begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\square \ \mathbf{A}_1^* = \left(\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right)$$

$$\square \mathbf{A}_1^* = \begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$$

Question 14 What is the nature of the equilibrium point (x_1^*, y_1^*) de (S_3) ?

- ☐ It is asymptotically stable
- ☐ It is unstable
- ☐ This is a saddle node

Equilibrium point (x_2^*, y_2^*) de (S_3) is non hyperbolic. Question 15

☐ True

Question 16 Among the following relationships, which ones are true?

$$r^2 = x^2 - y^2$$

$$\Box x = r \sin \theta \text{ et } y = r \cos \theta$$
 $\Box \tan \theta = \frac{x}{y}$

$$\Box \tan \theta = \frac{x}{u}$$

Let
$$(S_4)$$
:
$$\begin{cases} \dot{x} = x - xy \\ \dot{y} = -y + xy \end{cases}$$
.

Question 17 \clubsuit The jacobian matrix at the equlibrium point origin in system(S_4):

- always corresponds to a saddle node
- corresponds to centers
- \square is the matrix of the linear part of (S_4)

Question 18 \clubsuit Which ones of the following functions is a first integral for (S_4) ?

$$H(x,y) = \frac{x^2}{y} - \ln x - \ln y$$

$$\prod H(x,y) = -\ln x - \ln y + x + y$$

Question 19 🌲 Which condition(s) on H(x,y) is (are) required to demonstrate the existence of centers in (S_4) ?

- \square H(x,y) is a continuous function
- \square H(x,y) is an homeomorphism
- \square H(x,y) has closed level curves
- $\prod H(x,y)$ is constant along the trajectories

| Question 20 | The positive quadrant is positively invariant for (S_4) ? |
|-------------|---|
| □ No | ☐ Yes |