Dessine-moi un hibou

1.1 $f(x) = x\sqrt{1-\frac{x^2}{2}}$

1) D_f : $1-\frac{x^2}{2} \geqslant 0 \iff x^2 \leq 2$

$\iff -\sqrt{2} \leq x \leq \sqrt{2}$

$D_f = [-\sqrt{2} ; \sqrt{2}]$

2) $f(-x) = -f(x)$ impaire

↳ on peut l'étudier sur $[0 ; \sqrt{2}]$

3) $f(0) = 0$

$f(1) = 1/\sqrt{2}$

$f(\sqrt{2}) = 0$

4) $f'(x) = \sqrt{1-\frac{x^2}{2}} + \frac{x(-x)}{2\sqrt{1-x^2/2}}$

$$= \frac{1-x^2/2 - x^2/2}{\sqrt{1-x^2/2}} = \frac{1-x^2}{\sqrt{1-x^2/2}}$$

5) $f'(x) = 0 \iff 1-x^2 = 0$

$\iff x = -1, 1$

Deux extrema en $(1, 1/\sqrt{2})$ et $(-1, -1/\sqrt{2})$

6) $\lim\limits_{\substack{x \to \sqrt{2} \\ <}} f'(x) = -\infty$

$\Longrightarrow$ tangente verticale en $x = \sqrt{2}$

7)

x	0		1		$\sqrt{2}$
f'	1	$+$	0	$-$	$-\infty$
f	0	$\nearrow$	$1/\sqrt{2}$	$\searrow$	1

8) cf. graphe.

$$\begin{cases} \dot{x} = y \\ \dot{y} = x - x^3 \end{cases}$$

9) $\dot{x} = 0 \iff y = 0$

$\dot{y} = 0 \iff x(1-x^2) = 0$

$\iff x = 0, 1, -1$

$(0,0)$ $(-1,0)$ $(1,0)$

10) $J = \begin{pmatrix} 0 & 1 \\ 1-3x^2 & 0 \end{pmatrix}$

11) $J_{(0,0)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\det = -1$

P. S.

$J_{(-1,0)} = J_{(1,0)} = \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix}$ $\det = 2$, $\mathrm{Tr} = 0$

$\hookrightarrow$ la linéarisation prévoit des centres

12) $H(x,y) = 2x^2 - x^4 - 2y^2$

on veut montrer que $\frac{dH}{dt} = 0$ le long des trajectoires du système.

$$\frac{dH}{dt} = \frac{\partial H}{\partial x}\dot{x} + \frac{\partial H}{\partial y}\dot{y}$$

$$= (4x - 4x^3)y - 4y(x - x^3)$$

$$= 4xy - 4x^3y - 4xy + 4x^3y$$

$$= 0.$$

13) $H(x,y) = 0$

$$\Leftrightarrow \quad 2x^2 - x^4 - 2y^2 = 0$$

$$\Leftrightarrow \quad y^2 = x^2 - \frac{x^4}{2}$$

$$\Leftrightarrow \quad y^2 = x^2\left(1 - \frac{x^2}{2}\right)$$

$$\Leftrightarrow \quad y = \pm x\sqrt{1 - \frac{x^2}{2}} = \pm f(x).$$

14) cf. graphe.

15) $H(x,y) \simeq H(1,0)$

$$+ \left.\frac{\partial H}{\partial x}\right|_{(1,0)}(x-1) + \left.\frac{\partial H}{\partial y}\right|_{(1,0)} y$$

$$+ \frac{1}{2}\left.\frac{\partial^2 H}{\partial x^2}\right|_{(1,0)}(x-1)^2 + \frac{1}{2}\left.\frac{\partial^2 H}{\partial y^2}\right|_{(1,0)} y^2$$

$$+ \left.\frac{\partial^2 H}{\partial x \partial y}\right|_{(1,0)}(x-1)y$$

$$\frac{\partial H}{\partial x} = 4x - 4x^3 \qquad \left.\frac{\partial H}{\partial x}\right|_{(1,0)} = 0$$

$$\frac{\partial^2 H}{\partial x^2} = 4 - 12x^2 \qquad \left.\frac{\partial^2 H}{\partial x^2}\right|_{(1,0)} = -8$$

$$\frac{\partial^2 H}{\partial x \partial y} = 0$$

$$\frac{\partial H}{\partial y} = -4y \qquad \left.\frac{\partial H}{\partial y}\right|_{(1,0)} = 0$$

$$\frac{\partial^2 H}{\partial y^2} = -4$$

$$H(x,y) \simeq H(1,0) - 4(x-1)^2 - 2y^2$$

16) $H(x,y) - H(1,0) < 0$

extremum (maximum) local

$-4(x-1)^2 - 2y^2$ est de forme ellipsoïdale

↳ les courbes de niveaux se referment.

17) On en conclut qu'il y a effectivement des centres localement autour des points d'éq. $(-1,0)$ et $(1,0)$.

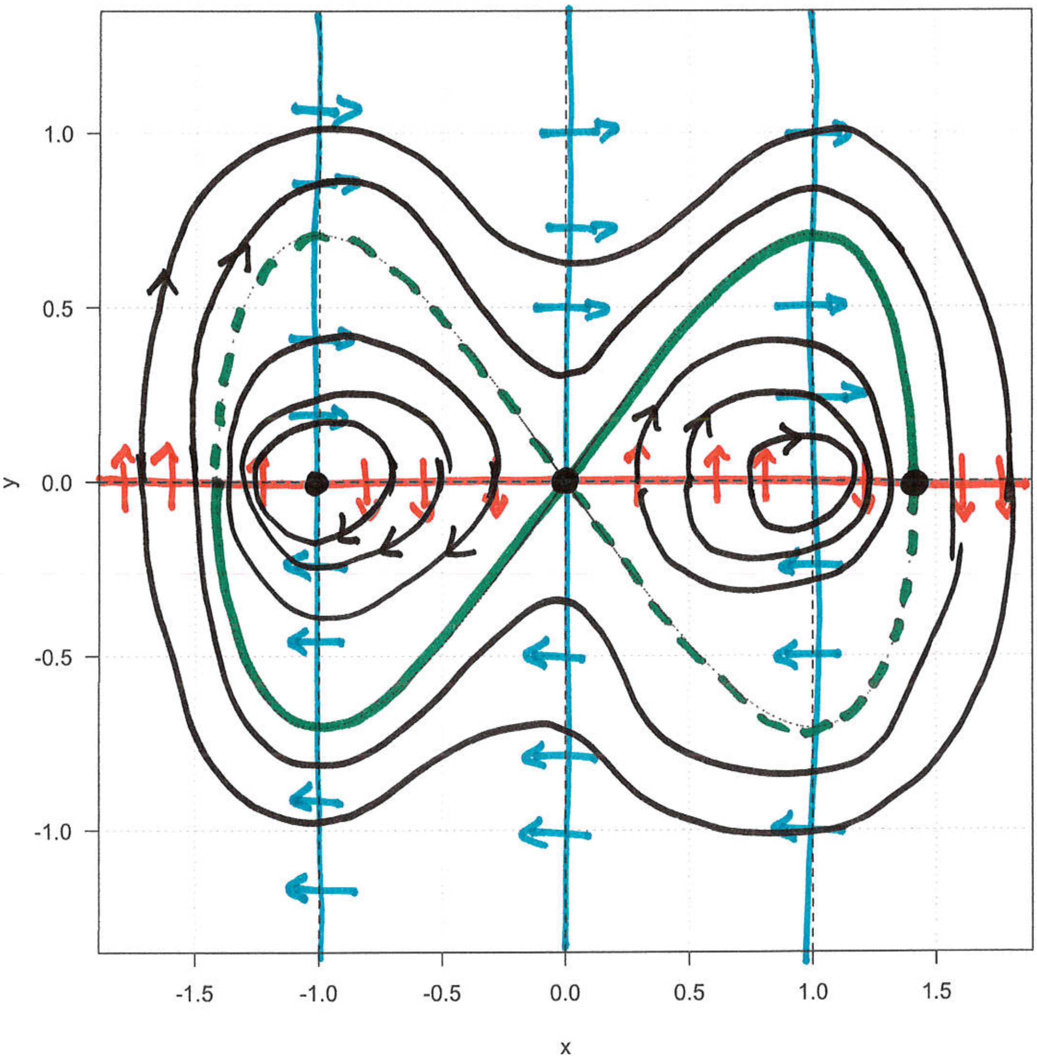
18) $\updownarrow$ $\quad \vec{v} \left| \begin{array}{l} 0 \\ \dot{y} \end{array} \right.$ $\quad \dot{x} = 0 \iff y = 0$

19) $\dot{y} = x(1-x^2) > 0$ si $0 < x < 1$

20) $\longleftrightarrow$ $\quad \vec{v} \left| \begin{array}{l} \dot{x} \\ 0 \end{array} \right.$ $\quad \dot{y} = 0 \iff x = 0, 1, -1$

21) $\dot{x} = y > 0$ si $y > 0$

22) cf. graphe



● : points d'équilibre

— : $f(x)$

— + - - - : $H(x,y) = 0$

— : isoclines verticale

— : isoclins horizontals

— : trajectoires

qui ?

$$\begin{cases} \dot{x}_1 = x_1 (r_1 - a_{11} x_1 - a_{12} x_2 - b_1 y_1) \\ \dot{x}_2 = x_2 (r_2 - a_{21} x_1 - a_{22} x_2 - b_2 y_2) \\ \dot{y}_1 = y_1 (-c_1 + d_1 x_1) \\ \dot{y}_2 = y_2 (-c_2 + d_2 x_2) \end{cases}$$

2proies / 2prédateurs ⊕ compétitio entre proies

$$\begin{cases} r_1 - a_{11} x_1 - a_{12} x_2 - b_1 y_1 = 0 \\ r_2 - a_{21} x_1 - a_{22} x_2 - b_2 y_2 = 0 \\ x_1^* = c_1 / d_1 \\ x_2^* = c_2 / d_2 \end{cases}$$

$$\rightarrow \begin{cases} y_1^* = \frac{1}{b_1}\left(r_1 - \frac{a_{11} c_1}{d_1} - \frac{a_{12} c_2}{d_2}\right) \\ y_2^* = \frac{1}{b_2}\left(r_2 - \frac{a_{21} c_1}{d_1} - \frac{a_{22} c_2}{d_2}\right) \end{cases}$$

<u>Cond°</u> $r_1 > a_{11} \frac{c_1}{d_1} + a_{12} \frac{c_2}{d_2}$

et $r_2 > a_{21} \frac{c_1}{d_1} + a_{22} \frac{c_2}{d_2}$

$$J = [j_{ij}]$$

$$j_{11} = r_1 - 2a_{11} x_1 - a_{12} x_2 - b_1 y_1$$

$$j_{12} = -a_{12} x_1$$

$$j_{13} = -b_1 x_1$$

$$j_{14} = 0$$

$j_{21} = -a_{21} x_2$

$j_{22} = r_2 - a_{21} x_1 - 2a_{22} x_2 - b_2 y_2$

$j_{23} = 0$

$j_{24} = -b_2 x_2$

$j_{31} = d_1 y_1$

$j_{32} = 0$

$j_{33} = -c_1$

$j_{34} = 0$

$j_{41} = 0$

$j_{42} = d_2 y_2$

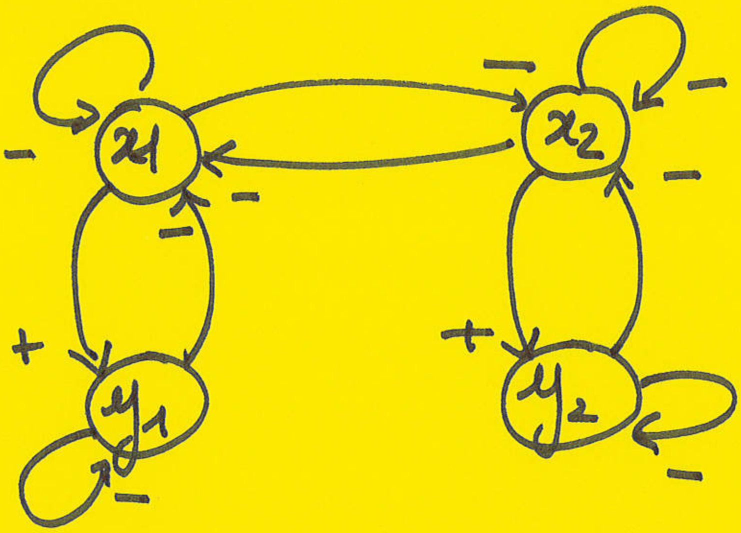
$j_{43} = 0$

$j_{44} = -c_2$

Au point d'équilibre non trivial :

$$J = \begin{pmatrix} -a_{11} x_1^* & -a_{12} x_1^* & -b_1 x_1^* & 0 \\ -a_{21} x_2^* & -a_{22} x_2^* & 0 & -b_2 x_2^* \\ d_1 y_1^* & 0 & -c_1 & 0 \\ 0 & d_2 y_2^* & 0 & -c_2 \end{pmatrix}$$

$$M = \begin{pmatrix} - & - & - & 0 \\ - & - & 0 & - \\ + & 0 & - & 0 \\ 0 & + & 0 & - \end{pmatrix}$$



Quirk-Ruppert

Condition 3. on a une relaθ −/−

↳ on ne peut pas conclure !