Exercise
(identifier : bifr-03)

Exercise (Bifurcation in $\mathbb{R}$) – statement

Let study the following ordinary differential equation :

$$\frac{dx(t)}{dt} = x + \frac{rx(t)}{1 + x(t)^2}$$

where r is a real constant ($r \in \mathbb{R}$).

1. Determine the number of equilibrium points according to the value of r.
2. Determine the stability of these equilibrium points according to the value of r.
3. Deduce the corresponding bifurcation diagram.
4. Deduce the type of bifurcation.
5. Draw some trajectories (chronicles) for appropriately chosen r values and different initial conditions.