# Exercise: Population dynamics - Classical models
(identifier: edor-TD)

## Population dynamics - Classical models (INSA-3BS-1.1) – statement

In order to model the dynamics of isolated populations, we consider different EDO-based models defined as follows:

$$\frac{dN(t)}{dt} = N(t)g(N(t))$$

where $N(t)$ is the population density and $g(N(t))$ the intrinsic population growth rate.

1. Demonstrate that an equilibrium point of this equation, denoted $N^* \neq 0$ is asymptotically stable if and only if $\left.\frac{dg}{dN}\right|_{N^*} < 0$.
2. Differents models have been proposed:
   (a) Gompertz model $g(N) = r\ln(K/N)$
   (b) Allee effect model $g(N) = r(K - N(t))(N(t) - \alpha)$
   (c) Allee effect model with exploitation $g(N) = r(K - N(t))(N(t) - \alpha) - E$ (with $\alpha < K$)

   Where parameters $r$, $K$, $\alpha$ and $E$ are strictly positive.

In each case, draw $g(N)$ as a function of $N$. Deduce then the equilibrium points and their stability.