Correction

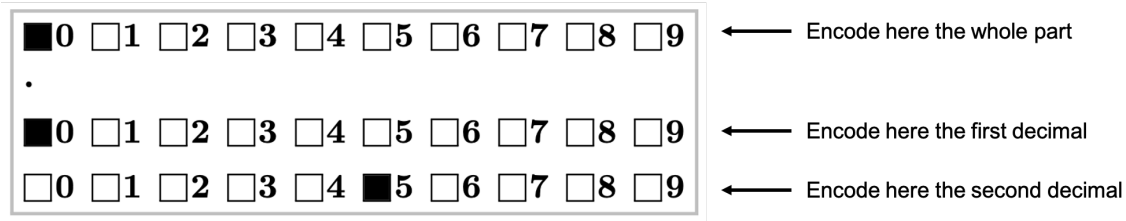
3BS - Biomathematics 1
Model examination – Duration: 3 hours

Instructions

This form will be scanned, so we ask you to strictly adhere to the rules below:

- To tick a box, fill it in black (■) by using black ball point pen;
- To correct, erase the box with white corrector ; **Do not redraw it**;
- Do not write anything in the header or margins of the pages;
- The symbol ♣ indicates that the number of correct answers is indeterminate (0, 1, 2,...). Its absence means that the question has aone single correct answer.

Below is the way to encode a numerical value :



Multiple-choice questions have a null expected value: right answer = 1 point; no answer = 0 point; wrong answer to a question with n proposals = $-\frac{1}{n-1}$ points.

If the examination takes place on site, you may bring an original hand-written double-sided A4 sheet of paper, the content of which being at your convenience, as well as any type of calculator **not connected to the Internet**. The use of mobile phones is strictly forbidden.

If the examination takes place at distance, you may use any document and/or material at your convenience.

Identity

Using your student card, fill in the fields below and encode your student number on the right side.

First and last names:

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Sutdent number:

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Preliminary questions

Question 1 ♣ Let $\frac{dx(t)}{dt} = 2x(t) - 3x^2(t)$. Is it:

□ a linear ODE?

■ an autonomous ODE?

■ an ODE of order 1?

Question 2 For an equilibrium point x^* of an ODE in $\mathbb{R}$ written as $\frac{dx(t)}{dt} = f(x(t))$, how many topological equivalence classes are there?

☐ 1 ■ 4 ☐ 3 ☐ 2

Question 3 ♣ What is the qualitative analysis of an ODE of the form $\frac{dx(t)}{dt} = f(x(t))$?

- ■ Graphically represent the shape of solutions
- ■ Study the stability of equilibrium points
- ☐ Exactly solve the ODE
- ■ Search for inflection points
- ■ Look for equilibrium points

Question 4 What is called chronicle when studying an ODE of the form $\frac{dx(t)}{dt} = f(x(t))$?

- ☐ The graphical representation of the phase portrait
- ☐ The location of the inflection points
- ☐ The study of the properties of function $f(x)$
- ☐ The search for stability of equilibrium points
- ■ The graphical representation of one of the solutions

Question 5 From the following equations, identify Malthus' model.

- ■ $\frac{dx(t)}{dt} = rx(t)$
- ☐ $\frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right)$
- ☐ $\frac{dx(t)}{dt} = rx(t)\ln\left(\frac{K}{x(t)}\right)$
- ☐ $\frac{dx(t)}{dt} = rx(t)\left(1 - \left(\frac{x(t)}{K}\right)^{\theta}\right)$

Question 6 From the following equations, identify the logistic model.

- ■ $\frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right)$
- ☐ $\frac{dx(t)}{dt} = rx(t)$
- ☐ $\frac{dx(t)}{dt} = rx(t)\ln\left(\frac{K}{x(t)}\right)$
- ☐ $\frac{dx(t)}{dt} = rx(t)\left(1 - \left(\frac{x(t)}{K}\right)^{\theta}\right)$

Question 7 From the following equations, identify the Gompertz model.

- ☐ $\frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right)$
- ☐ $\frac{dx(t)}{dt} = rx(t)$
- ■ $\frac{dx(t)}{dt} = rx(t)\ln\left(\frac{K}{x(t)}\right)$
- ☐ $\frac{dx(t)}{dt} = rx(t)\left(1 - \left(\frac{x(t)}{K}\right)^{\theta}\right)$

Question 8 ♣ Which equation is solved when looking for equilibrium points of an equation such as $\frac{dx(t)}{dt} = f(x(t))$?

- ☐ $\frac{df(x(t))}{dx} = 0$
- ☐ $\frac{d^2x(t)}{dt^2} = 0$
- ■ $f(x(t)) = 0$
- ■ $\frac{dx(t)}{dt} = 0$

Question 9 ♣ How do we study the stability of equilibrium points x^* of an equation such as $\frac{dx(t)}{dt} = f(x(t))$?

- ■ By studying the sign of $f(x(t))$
- ■ By linearising $f(x(t))$ in the neighboorhood of x^*
- ☐ By studying the sign of $\frac{d^2x(t)}{dt^2}$
- ■ By studying the sign of $\frac{dx(t)}{dt}$
- ■ By studying the variations in the solutions of the equation

Question 10 ♣ Which equation is solved when looking for the inflection points of an equation such as $\frac{dx(t)}{dt} = f(x(t))$?

- ☐ $f(x(t)) = 0$
- ☐ $\frac{dx(t)}{dt} = 0$
- ■ $\frac{d^2x(t)}{dt^2} = 0$
- ■ $\frac{df(x)}{dx} = 0$

The following five questions explore the following equation (E_1) : $\frac{dx(t)}{dt} = x(t) - x^2(t)$.

Question 11 Identify equilibrium points of (E_1).

- ☐ $x_1^* = -1$ et $x_2^* = 1$
- ■ $x_1^* = 0$ et $x_2^* = 1$
- ☐ $x_1^* = 0$ et $x_2^* = -1$
- ☐ $x_1^* = 1$ et $x_2^* = 2$

Question 12 To which topological equivalence class does x_1^* belong?

- ■ Unstable
- ☐ Asymptotically stable
- ☐ Positive shunt
- ☐ Negative shunt

Question 13 To which topological equivalence class does x_2^* belong?

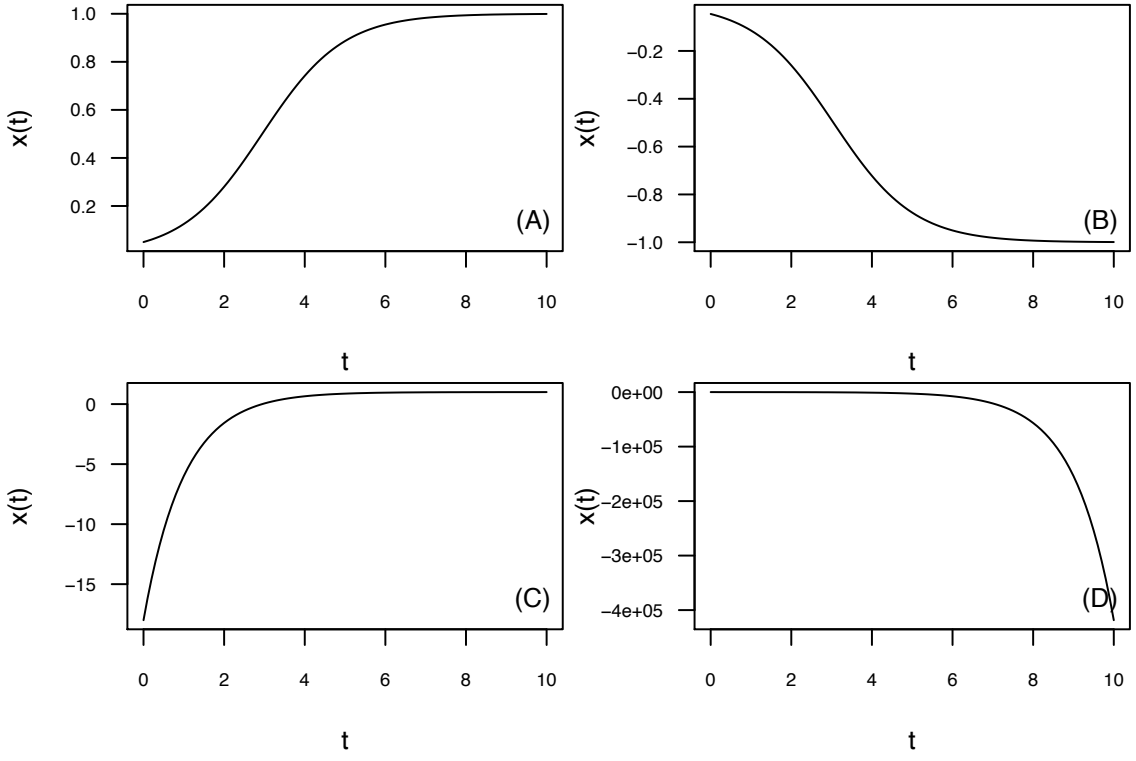
- ☐ Unstable
- ☐ Negative shunt
- ☐ Positive shunt
- ■ Asymptotically stable

Question 14 Does the equation (E_1) admit an inflection point?

- ■ yes
- ☐ no

Question 15 Which of the chronicles below corresponds to (E_1) for $x(0)$ =0.05?

■ (A) ☐ (D) ☐ (C) ☐ (B)



Question 16 ♣ Which of the following proposals correspond to linear systems?

■ $\begin{cases} \frac{dx(t)}{dt} = x + 2y \\ \frac{dy(t)}{dt} = 3x + 4y \end{cases}$ ☐ $\begin{cases} \frac{dx(t)}{dt} = x - xy \\ \frac{dy(t)}{dt} = -y + xy \end{cases}$

☐ $\begin{cases} \frac{dx(t)}{dt} = x(1 - 2y) \\ \frac{dy(t)}{dt} = y(1 - 2x) \end{cases}$ ■ $\begin{cases} \frac{dx(t)}{dt} = x - y \\ \frac{dy(t)}{dt} = y - x \end{cases}$

The following four questions consider the following system (S_1) : $\begin{cases} \frac{dx(t)}{dt} = x - y \\ \frac{dy(t)}{dt} = y + x \end{cases}$

Question 17 What is the matrix associated with (S_1)?

☐ $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ ☐ $\mathbf{A} = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$

■ $\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ ☐ $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Question 18 What is the characteristic equation of the matrix associated with (S_1)?

☐ $\lambda^2 - 2\lambda - 2 = 0$ ☐ $\lambda^2 + 2\lambda - 2 = 0$

■ $\lambda^2 - 2\lambda + 2 = 0$ ☐ $\lambda^2 + 2\lambda + 2 = 0$

Question 19 What are the eigenvalues of the matrix associated with (S_1)?

☐ $\lambda_1 = 0$ et $\lambda_2 = 1$ ☐ $\lambda_1 = 1$ et $\lambda_2 = -1$

☐ $\lambda_1 = i$ et $\lambda_2 = -i$ ■ $\lambda_1 = 1 + i$ et $\lambda_2 = 1 - i$

Question 20 What can be deduced from the eigenvalues of the matrix associated with (S_1)?

☐ The equilibrium point $(0, 0)$ is an asymptotically stable spiral

☐ The equilibrium point $(0, 0)$ is an unstable node

☐ The equilibrium point $(0, 0)$ is a saddle node

■ The equilibrium point $(0, 0)$ is an unstable spiral

Question 21 ♣ Let $\dot{\mathbf{X}} = \mathbf{AX}$ with $\mathbf{A}$ a matrix of dimension 2 such that $\det(\mathbf{A}) \neq \mathbf{0}$. Under which condition(s) can we say that the equilibrium point $\mathbf{X}^*$ is asymptotically stable?

■ $\mathrm{tr}(\mathbf{A}) < \mathbf{0}$ ■ $\det(\mathbf{A}) > \mathbf{0}$ ☐ $\Delta > 0$

☐ $\det(\mathbf{A}) < \mathbf{0}$ ☐ $\Delta < 0$ ☐ $\mathrm{tr}(\mathbf{A}) > \mathbf{0}$

Question 22 How many are real Jordan's matrix forms associated with a square matrix of dimension 2?

☐ 3 ☐ 2 ☐ 1 ■ 4

Question 23 ♣ Which of the following matrices are in the form of Jordan?

☐ $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ ■ $\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix}$ ■ $\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$

☐ $\begin{pmatrix} 3 & 1 \\ 0 & 4 \end{pmatrix}$ ☐ $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ ■ $\begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}$

Question 24 Which of the following matrices corresponds to an origin equilibrium point that is an unstable spiral?

☐ $\begin{pmatrix} 3 & 1 \\ 0 & 4 \end{pmatrix}$ ☐ $\begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}$ ☐ $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$

☐ $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ ☐ $\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$ ■ $\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix}$

Question 25 Which of the following matrices corresponds to an origin equilibrium point which is an asymptotically stable degenerated node?

☐ $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ ☐ $\begin{pmatrix} 3 & 1 \\ 0 & 4 \end{pmatrix}$ ☐ $\begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}$

☐ $\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix}$ ☐ $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ ■ $\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$

The following questions examine the following system (S_2) : $\begin{cases} \frac{dx(t)}{dt} = x(t) - x(t)y(t) \\ \frac{dy(t)}{dt} = -y(t) + x(t)y(t) \end{cases}$

Question 26 What ecological interaction does (S_2) model?

■ prrey-predator ☐ symbiosis ☐ competition ☐ commensalism

Question 27 Provide equilibrium points for (S_2).

☐ $(x_1^*, y_1^*) = (0,0)$ et $(x_2^*, y_2^*) = (1,-1)$ ☐ $(x_1^*, y_1^*) = (0,1)$ et $(x_2^*, y_2^*) = (0,1)$

☐ $(x_1^*, y_1^*) = (0,1)$ et $(x_2^*, y_2^*) = (1,1)$ ■ $(x_1^*, y_1^*) = (0,0)$ et $(x_2^*, y_2^*) = (1,1)$

Question 28 What is the Jacobian matrix associated with (S_2)?

■ $\mathbf{A} = \begin{pmatrix} 1-y & -x \\ y & x-1 \end{pmatrix}$ ☐ $\mathbf{A} = \begin{pmatrix} 1-y & y \\ -x & x-1 \end{pmatrix}$

☐ $\mathbf{A} = \begin{pmatrix} 1-y & x \\ -y & x-1 \end{pmatrix}$ ☐ $\mathbf{A} = \begin{pmatrix} x-1 & -x \\ y & 1-y \end{pmatrix}$

Question 29 What is the Jacobian matrix from (S_2) to equilibrium point (x_2^*, y_2^*)?

☐ $\mathbf{A}_2^* = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ ■ $\mathbf{A}_2^* = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

☐ $\mathbf{A}_2^* = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ ☐ $\mathbf{A}_2^* = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Question 30 What can be said about the nature of equilibrium point (x_2^*, y_2^*)?

☐ The equilibrium point is unstable

☐ The equilibrium point corresponds to centres

☐ The equilibrium point is asymptotically stable.

■ Linearisation foresees centres

Case study

We are now interested in the dynamics of a rabbit population (of density $L(t)$) in interaction with another species (of density $R(t)$). Both populations are located within the same area where rabbit hunting is allowed (through parameter c). The combined dynamics of both species is then described by the following model:

$$(S_3) \quad \begin{cases} \frac{dL(t)}{dt} = rL(t)\ln\left(\frac{K}{L(t)}\right) - cL(t) - aL(t)R(t) \\ \frac{dR(t)}{dt} = eaL(t)R(t) - mR(t) \end{cases}$$

where parameters r, K, c, a, e and m are real and strictly positive.

Part 1

In this first part, we will consider that second species (variable $R(t)$) is absent: $\forall t\ R(t) = 0$. The rabbit population dynamics is then described with the following equation:

$$\frac{dL(t)}{dt} = rL(t)\ln\left(\frac{K}{L(t)}\right) - cL(t) = f(L(t)) \quad (E_1)$$

Question 31 If parameter $c = 0$, which growth model describes the rabbit population dynamics?

- ☐ Weibull
- ☐ Malthus
- ☐ Verhuslt
- ■ Gompertz

Question 32 When parameter $c \neq 0$, what type of exploitation is it?

- ☐ other
- ☐ constant
- ☐ non linear
- ■ linear

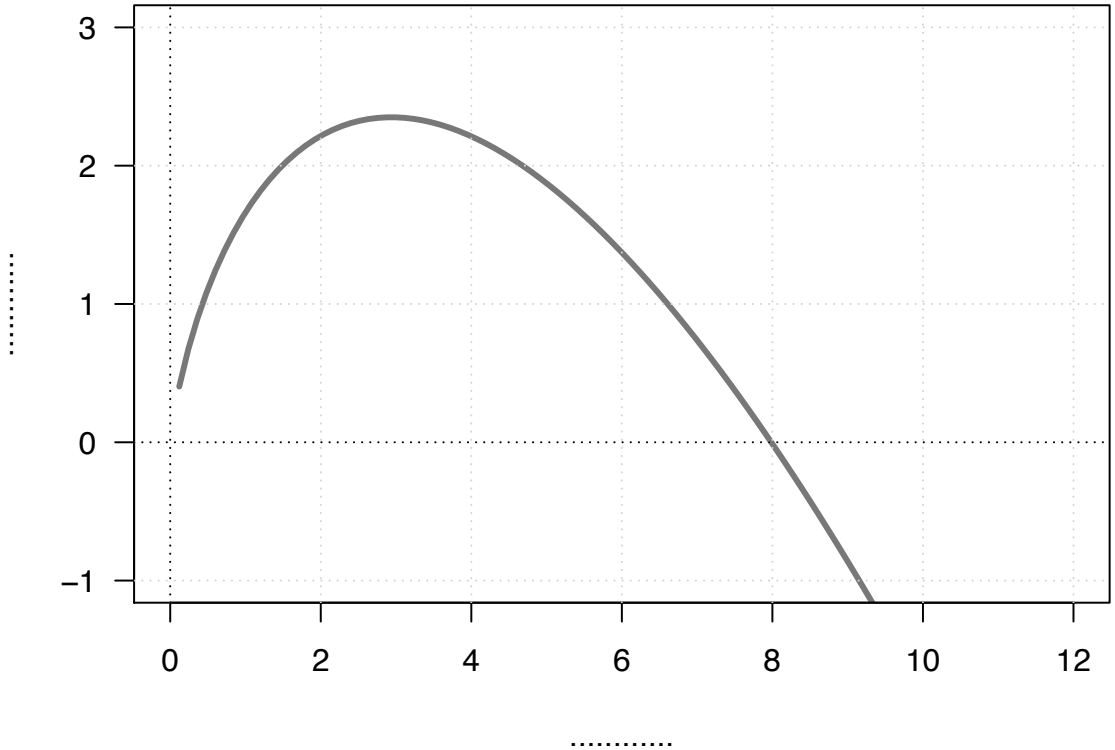
Question 33 What is the meaning of parameter c?

- ☐ carrying capacity
- ☐ growth rate
- ■ exploitation rate
- ☐ mortality rate

Question 34 Which R function may be used to draw function $f(L)$ as a function of L?

- ☐ `phaseR`
- ☐ `trajectory`
- ☐ `flowField`
- ■ `curve`
- ☐ `abline`
- ☐ `nullclines`

```
> r <- 0.8
> K <- 10
> c <- ...
> .....(r * x * log(K / x) - c * x, from = 0, to = 150,
       xlab = ".............", ylab = ".............",
       col = "...", lwd = 2, las = 1)
> abline(h = 0, lty = 3)
> abline(v = 0, lty = 3)
```



Question 35 Given the previous R code and its graphical output above, what is the approximate numerical value of parameter c?

■0 □1 □2 □3 □4 □5 □6 □7 □8 □9
.
□0 ■1 □2 □3 □4 □5 □6 □7 □8 □9
□0 □1 □2 □3 □4 □5 □6 □7 ■8 □9

Question 36 What is the appropriate x-axis label of the previous figure?

□ t □ $f(L)$ ■ L □ $L(t)$

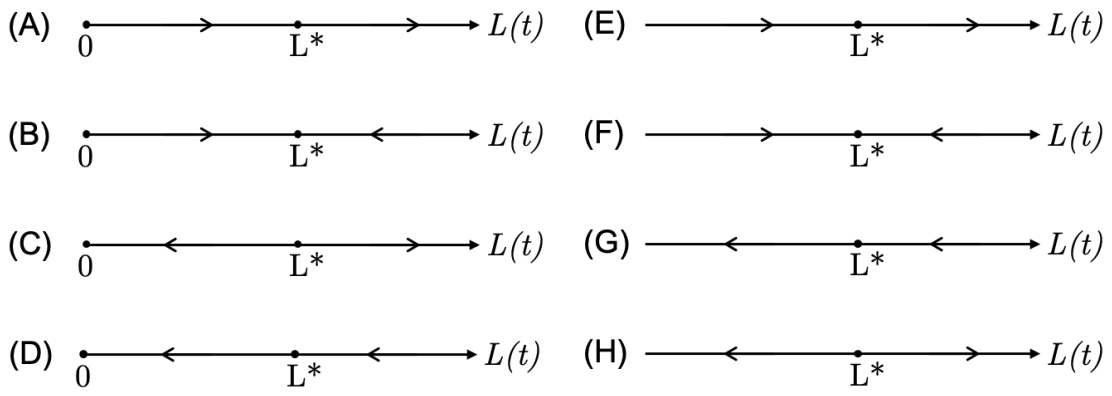
Question 37 How many equilibrium points has equation (E_1)?

□ none □ 2 □ 0 ■ 1

Question 38 What is the stability of the non null equilibrium point?

■ Asymptotically stable □ Unstable
□ Negative shunt □ Positive shunt

Question 39 What is the phase portrait associated to equation (E_1)?



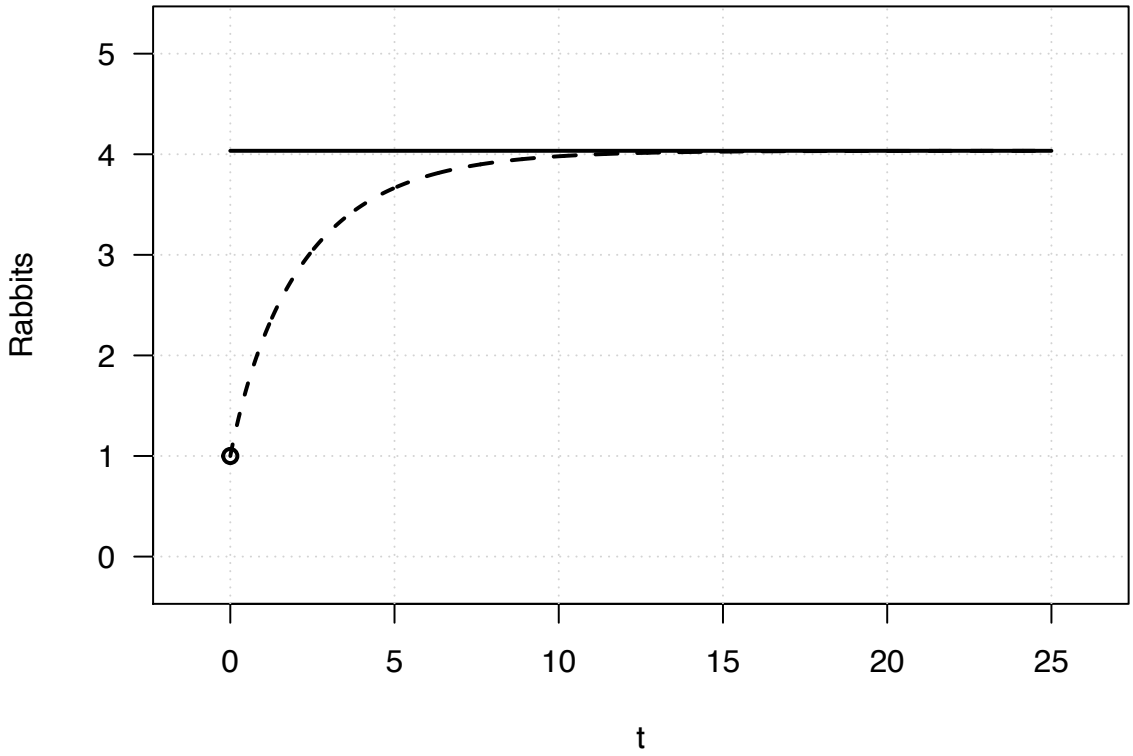
☐ (E) ☐ (C) ☐ (D) ■ (F) ☐ (B) ☐ (H) ☐ (A) ☐ (G)

Question 40 What is the correct R-package to perform numerical simulations of equation (E_1)?

☐ `flowField` ☐ `abline`
☐ `nullclines` ☐ `curve`
☐ `trajectory` ■ `phaseR`

Question 41 What is the appropriate R function to draw chronicles of equation (E_1)?

☐ `curve` ☐ `flowField`
■ `trajectory` ☐ `phaseR`
☐ `abline` ☐ `nullclines`



Question 42 According to the previous figure, what is the initial condition $L(0)$?

☐0 ■1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9
.
■0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9

Question 43 Does the chronicle on the above figure show an inflexion point?

■ no ☐ yes

Question 44 On the figure above, would it be possible to draw chronicles exhibiting an inflexion point?

☐ no ■ yes

Question 45 If you answered `yes` to the previous question, what is the inflexion point? If you answered `no`, confirm your decision.

☐ I confirm there is no inflexion point for equation (E_1)

■ $L_i = Ke^{-\frac{r+c}{r}}$

☐ $L_i = \frac{K}{2}$

☐ $L_i = Ke^{-\frac{c}{r}}$

Part 2

We will now consider the combined dynamics of both populations (densities $L(t)$ and $R(t)$) according to the above system (S_3) but assuming that now hunting is forbidden. Hence, parameter $c = 0$. System (S_3) becomes now system (S_4) writing as follows:

$$(S_4) \quad \begin{cases} \frac{dL(t)}{dt} = rL(t)\ln\left(\frac{K}{L(t)}\right) - aL(t)R(t) \\ \frac{dR(t)}{dt} = eaL(t)R(t) - mR(t) \end{cases}$$

Question 46 Which type of interactions is described by the above model?

■ predation ☐ symbiosise ☐ commensalism
☐ amensalism ☐ mutualism ☐ competition

Question 47 In this interaction, what role does the species associated with variable $R(t)$ play?

☐ prey ■ predator
☐ symbiote ☐ competitor

Question 48 What is the meaning of parameter a?

☐ competition rate ■ predation rate
☐ growth rate ☐ mortality rate

Question 49 What is the unit of parameter a? (# means 'number')

☐ $[t]^{-1}$ ☐ # ☐ t ■ $\#^{-1}.[t]^{-1}$

We choose the phase plane (L, R).

Question 50 What are the coordinates of vectors defining the flow field?

☐ $\vec{v} = (\frac{dR(t)}{dt}, \frac{dL(t)}{dt})$ ■ $\vec{v} = (\frac{dL(t)}{dt}, \frac{dR(t)}{dt})$

Question 51 Among the following proposals, identify which ones correspond to the equations of vertical nullclines.

- ☐ $L = 0$
- ■ $R = \frac{r}{a} \ln\left(\frac{K}{L}\right)$
- ☐ $L = \frac{m}{ea}$
- ☐ $R = 0$

Question 52 ♣ Among the following proposals, identify which ones correspond to the equations of horizontal nullclines.

- ☐ $L = 0$
- ☐ $R = \frac{r}{a} \ln\left(\frac{K}{L}\right)$
- ■ $L = \frac{m}{ea}$
- ■ $R = 0$

Question 53 Which R function can be used to draw the flow field of a dynamical system?

- ■ `flowField`
- ☐ `curve`
- ☐ `nullclines`
- ☐ `abline`
- ☐ `phaseR`
- ☐ `trajectory`

Question 54 Which R function can be used to draw nullclines of a dynamical system?

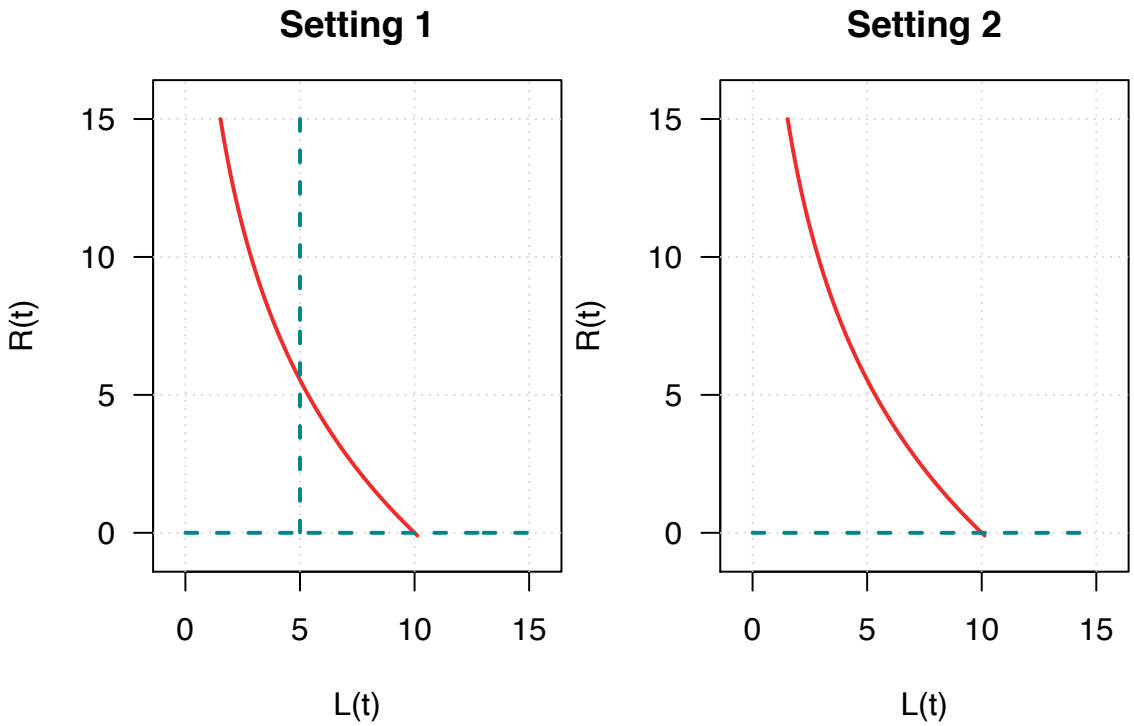
- ☐ `trajectory`
- ☐ `phaseR`
- ■ `nullclines`
- ☐ `flowField`
- ☐ `curve`
- ☐ `abline`

Question 55 In the R code lines below that assign system (S_4) to object `model`, to which set of parameters corresponds object `parameters`?

```
> model <- function(t, y, parameters)
 {
   dy <- numeric(2)
   dy[1] <- parameters[1] * y[1] * log(parameters[2] / y[1]) - parameters[3] *y[1] * y[2]
   dy[2] <- parameters[4] * parameters[3] *y[1] * y[2] - parameters[5] * y[2]
   list(dy)
 }
```

- ■ (r, K, a, e, m)
- ☐ (K, r, e, a, m)
- ☐ (e, a, m, r, K)
- ☐ (a, K, m, r, e)

Depending on parameter values, two graphical settings are possible.



Question 56 What color (or point type) are vertical nullclines?

☐ green (dotted) ■ red (plain)

Question 57 According to the previous graphs, what is the value of parameter K?

☐0 ■1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9
■0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9

Question 58 Assuming that $e = 0.5$ and $m = 0.25$ in setting 1, what is the value of parameter a?

■0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9
.
☐0 ■1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9

Question 59 ♣ Among the following coordinates, identify those that correspond to equilibrium points in Setting 1.

☐ $(0, K)$ ☐ $(0, 0)$ ■ $(\frac{m}{ea}, \frac{r}{a} \ln\left(\frac{eaK}{m}\right))$
☐ $(\frac{m}{ea}, \frac{r}{a} \ln\left(\frac{mK}{ea}\right))$ ☐ $(\frac{ea}{m}, \frac{r}{a} \ln\left(\frac{eaK}{m}\right))$ ■ $(K, 0)$

Question 60 Which condition on parameters leads to Setting 2?

■ $K < \frac{m}{ea}$ ☐ $K > \frac{m}{ea}$
☐ $K < \frac{e}{ma}$ ☐ $K > \frac{ea}{m}$

Question 61 How does the Jacobian matrix of system (S_4) write?

☐ $\mathbf{A} = \begin{pmatrix} -aL & r\left(\ln\left(\frac{K}{L}\right) - 1\right) - aR \\ eaL - m & eaR \end{pmatrix}$
☐ $\mathbf{A} = \begin{pmatrix} r\left(\ln\left(\frac{K}{L}\right) - 1\right) - aR & -aL \\ eaL - m & eaR \end{pmatrix}$
☐ $\mathbf{A} = \begin{pmatrix} -aL & r\left(\ln\left(\frac{K}{L}\right) - 1\right) - aR \\ eaR & eaL - m \end{pmatrix}$
■ $\mathbf{A} = \begin{pmatrix} r\left(\ln\left(\frac{K}{L}\right) - 1\right) - aR & -aL \\ eaR & eaL - m \end{pmatrix}$

Question 62 How does the Jacobian matrix of system (S_4) write when one of the species disappears?

■ $\mathbf{A_0} = \begin{pmatrix} -r & -aK \\ 0 & eaK - m \end{pmatrix}$
☐ $\mathbf{A_0} = \begin{pmatrix} -r & -aK \\ eaK - m & 0 \end{pmatrix}$
☐ $\mathbf{A_0} = \begin{pmatrix} -aK & -r \\ eaK - m & 0 \end{pmatrix}$
☐ $\mathbf{A_0} = \begin{pmatrix} -aK & -r \\ 0 & eaK - m \end{pmatrix}$

Question 63 ♣ What are the eigenvalues of the above matrix $\mathbf{A_0}$?

☐ $\lambda = 0$
☐ $\lambda = m - eaK$
■ $\lambda = -r$
■ $\lambda = eaK - m$
☐ $\lambda = -\frac{m}{e}$
☐ $\lambda = er\frac{eaK}{m}$

Question 64 Assuming that $K > \frac{m}{ea}$, what is the classification of the equilibrium point corresponding to matrix $\mathbf{A_0}$?

☐ Centers
☐ Asymptotically stable node
■ Saddle node
☐ Unstable spiral

Question 65 How does the Jacobian matrix of system (S_4) write when both species can coexist?

☐ $\mathbf{A}^* = \begin{pmatrix} -\frac{m}{e} & -r \\ er\ln\left(\frac{eaK}{m}\right) & 0 \end{pmatrix}$
☐ $\mathbf{A}^* = \begin{pmatrix} -r & -\frac{m}{e} \\ 0 & er\ln\left(\frac{eaK}{m}\right) \end{pmatrix}$
■ $\mathbf{A}^* = \begin{pmatrix} -r & -\frac{m}{e} \\ er\ln\left(\frac{eaK}{m}\right) & 0 \end{pmatrix}$
☐ $\mathbf{A}^* = \begin{pmatrix} -\frac{m}{e} & -r \\ 0 & er\ln\left(\frac{eaK}{m}\right) \end{pmatrix}$

Question 66 How does the determinant of matrix $\mathbf{A}^*$ write?

☐ $\det \mathbf{A}^* = er\ln\left(\frac{eaK}{m}\right)$
☐ $\det \mathbf{A}^* = -\frac{m}{e}$
☐ $\det \mathbf{A}^* = aK$
■ $\det \mathbf{A}^* = mr\ln\left(\frac{eaK}{m}\right)$

Question 67 How does the trace of matrix $\mathbf{A}^*$ write?

☐ $\mathrm{tr}(\mathbf{A}^*) = aK$
■ $\mathrm{tr}(\mathbf{A}^*) = -r$
☐ $\mathrm{tr}(\mathbf{A}^*) = -\frac{m}{e}$
☐ $\mathrm{tr}(\mathbf{A}^*) = 0$

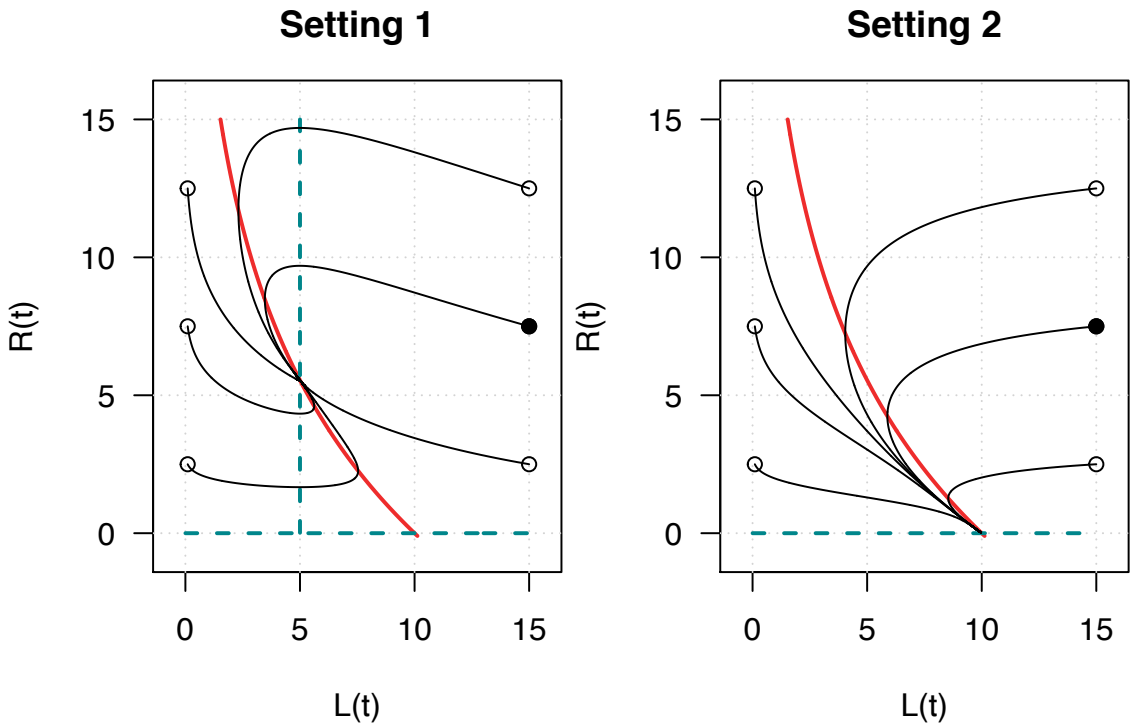
Question 68 Assuming that $K > \frac{m}{ea}$, what is the classification of the equilibrium point corresponding to the coexistence of both species?

☐ Unstable
■ Asymptotically stable
☐ Centers
☐ Saddle node

Question 69 Which R function could be used to get both coordinates and classification of equilibrium points?

☐ `stability`
☐ `flowField`
☐ `nullclines`
☐ `curve`
■ `findEquilibrium`
☐ `numericalSolution`

Below are phase portraits for the both settings.

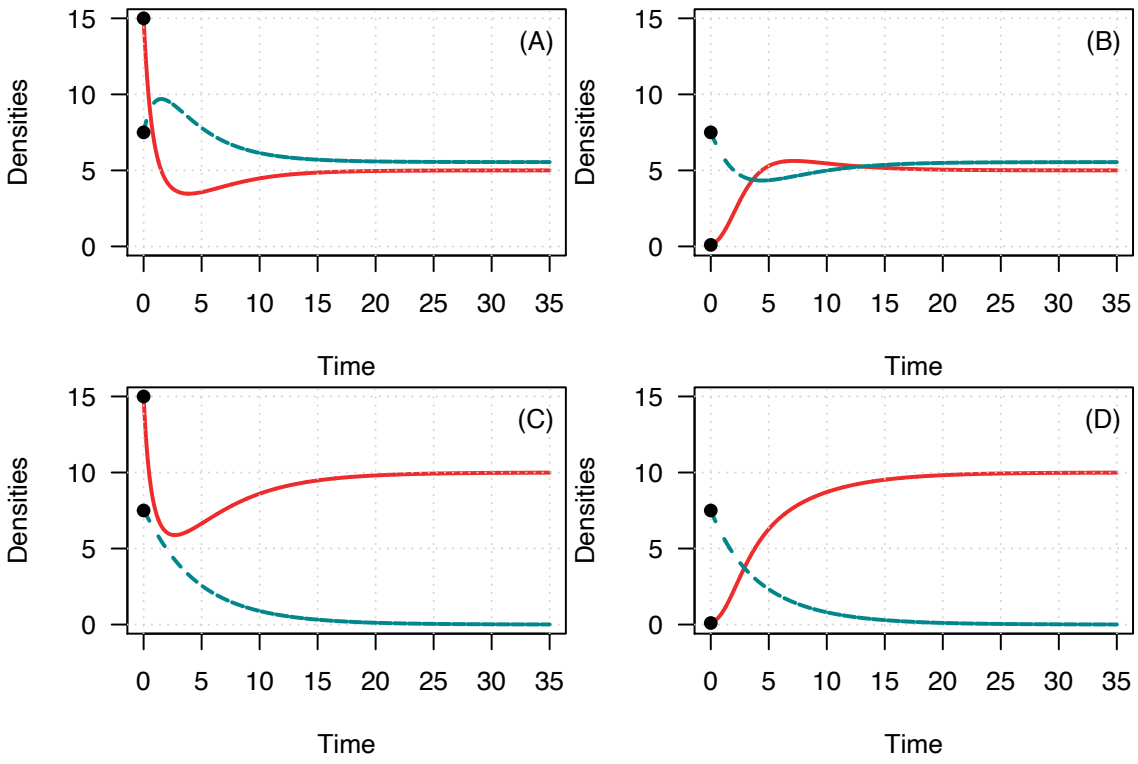


Question 70 From the figures below, identify which chronicles correspond to the trajectory starting from the black dot on the phase portrait of setting 1?

☐ (D) ☐ (C) ☐ (B) ■ (A)

Question 71 From the figures below, identify which chronicles correspond to the trajectory starting from the black dot on the phase portrait of setting 2?

☐ (B) ■ (C) ☐ (D) ☐ (A)



Question 72 In setting 1, what color (or point type) is the rabbit chronicle?

■ red (plain) ☐ green (dotted)

Question 73 In setting 2, what color (or point type) is the rabbit chronicle?

☐ green (dotted) ■ red (plain)