Fiche TD avec le logiciel R : TP2-English

INSA 4-BIM
TP2 : Systems of recursion equations

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Table des matières

1 Introduction

In this second session, we will study systems of (linear and non linear) recurrence equations.

2 System of linear recursion equations : propagation of an annual plant

In this part, we are interested in the propagation of an annual plant, *i.e*, which blooms only once and disperses its seeds before disappearing. We assume that a plant at year n emits an average number γ of seeds in August. After the first winter, only a fraction σ survives the winter and among these surviving seeds, only a fraction α will germinate the following spring to give plants at year $n+1$. The remaining seeds, which have not germinated, will spend the next winter with probability σ of surviving but only a fraction β will succeed in germinating after two years in the ground (year $n+2$). Those that have not germinated are considered dead. It is also assumed that a seed will never germinate after two years in the ground. This germination cycle is illustrated in Figure 1.

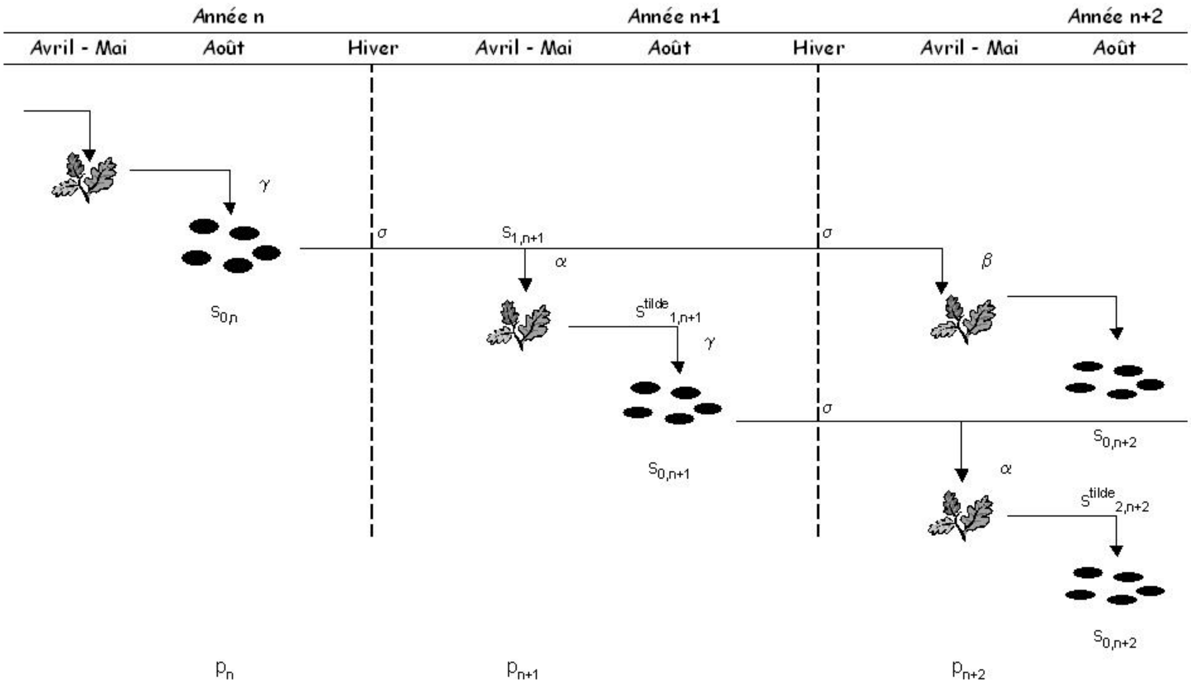


FIGURE 1 – Life cycle of the plant

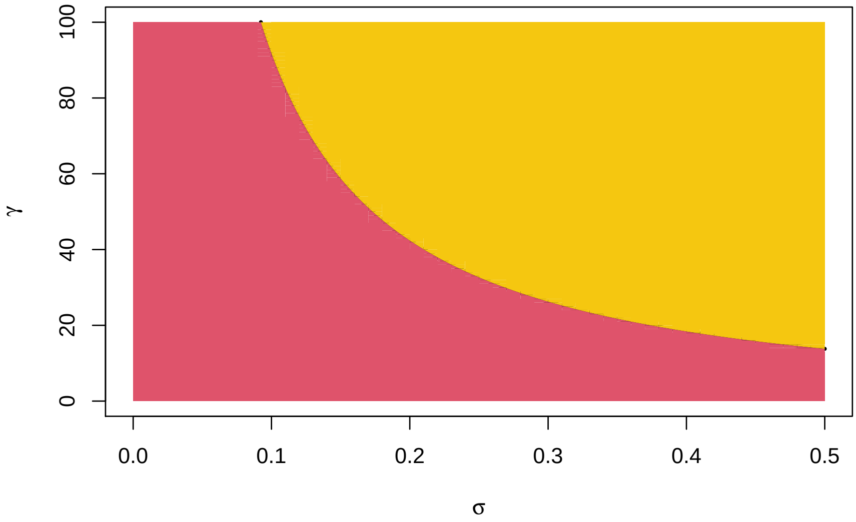
1. Determine the recurrence equation that gives the quantity of plants at year $n+2$, denoted p_{n+2}, as a function of p_{n+1} and p_n.
2. Create a function that calculates p_n. Apply this function to calculate the ten first values of p_n for $p_1 = 50$, $p_2 = 100$, $\gamma = 100$, $\alpha = 0.1$, $\beta = 0.1$ and $\sigma = 0.2$. You can use a recursive function.
3. Use the previous function to calculate the ten first values of the population growth rate p_{n+1}/p_n. What can you deduce concerning the long term population behaviour for this set of parameter values ?
4. Re-do the two previous questions for $\gamma = 30$. Discuss.
5. Is it possible to predict the value of the population growth rate when n tends to infinity, whitout explicitly calculating it as previsouly ?

6. Find the value of γ (with two significant digits) that makes stabilizing the population.

7. Compare your value with the theoretical one :

$$\gamma = \frac{1}{\alpha\sigma + \beta\sigma^2(1-\alpha)}$$

8. Reproduce the graph below by separating the areas where the values of γ and σ lead to the extinction of the population from areas that lead to population growth. Keep $\alpha = 0.1$ and $\beta = 0.1$. You will need to use the `contour` and `filled.contour` functions. Specify the legend.



3 System of non linear recursion equations in $\mathbb{R}^2$: host-paraisoid interactions

3.1 The Nicholson-Bailey model

In general, the interaction between a host population, H, and a parasitoid population, P is written :

$$\begin{cases} H_{t+1} = rH_t f(H_t, P_t) \\ P_{t+1} = cH_t[1 - f(H_t, P_t)] \end{cases}$$

where H_t is the host density (females) at time t, P_t is the parasitoid density at time t, r is the net reproduction rate of female hosts, c is the fecundity rate of parasitoids (mean number of parasitoids emerging from a host then surviving until the next time step) and $f(H_t, P_t)$ is the non parasitized host fraction.
In the Nicholson-Bailey model, $f(H_t, P_t) = e^{-aP_t}$.

1. Create a function that calculates the host and parasitoid densities over time according to this model.

2. Fix $r = 2$, $c = 1$, $a = 0.068$, $H_1 = 15$ and $P_1 = 10$. Give values of H_t and P_t at $t = 5$, 10, 15 and 20.

3. In the same graph, plot the dynamics of H_t and P_t for t from 0 to 20. Conclude.

4. Determine the analytical expression of Nicholson-Bailey model fixed points, as well as their stability in the general case where r, c, a, H_1 and P_1 are unknown.

5. Verify your conclusions for $r = 2$, $c = 1$, $a = 0.068$, $H_1 = 15$ and $P_1 = 10$. Are they consistent with what you observed in Question 3 ? Justify.

3.2 The Nicholson-Bailey model modified by Beddington *et al.*

Beddington *et al.* have modified the Nicholson-Bailey model to consider a logistic growth for hosts whitout parasitoids :

$$\begin{cases} H_{t+1} = e^{r(1-\frac{H_t}{K})} H_t f(H_t, P_t) \\ P_{t+1} = cH_t[1 - f(H_t, P_t)] \end{cases}$$

We always have $f(H_t, P_t) = e^{-aP_t}$.

1. Create a function that calculates the host and parasitoid densities over time according to this model.

2. Fix $r = 1$, $c = 1$, $K = 100$, $H_1 = 40$ and $P_1 = 30$. In the same graph, plot the dynamics of H_t and P_t for t from 0 to 50 and $a = 0.02$ or $a = 0.035$. Discuss.

3. Plot the corresponding phase plan ($P(t)$ according to $H(t)$).

4. Study the host population dynamics whitout parasitoids (fixed points, stability, bifurcation diagram).